

$\frac{12}{21} = \frac{4}{7}$, $\therefore \frac{AC}{EC} = \frac{4}{3}$. $\because AF \perp BC, EG \perp BC$,
 $\therefore \angle CFA = \angle CGE = 90^\circ$. 又 $\because \angle ECG = \angle ACF$, $\therefore \triangle ACF \sim \triangle ECG$, $\therefore \frac{AF}{EG} = \frac{AC}{EC}$, 即
 $\frac{AF}{6} = \frac{4}{3}$, 解得 $AF = 8$, $\therefore$ 河的宽度 AF 为 8 米.

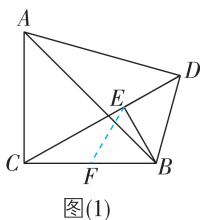
13. (1) 【证明】 $\because \triangle ABC$ 是等腰直角三角形,
 $BC = 2, \angle ACB = 90^\circ, \therefore AC = 2, \angle ABC = 45^\circ$,
 $\therefore AB = 2\sqrt{2}, \therefore \frac{BC}{AB} = \frac{\sqrt{2}}{2}$.

同理可得 $\angle DBE = 45^\circ, \frac{BE}{BD} = \frac{\sqrt{2}}{2}, \therefore \frac{BC}{AB} = \frac{BE}{BD}$.
 又 $\because \angle ABC = \angle EBD, \therefore \angle ABC - \angle ABE = \angle DBE - \angle ABE$,
 $\therefore \angle CBE = \angle ABD$,
 $\therefore \triangle ADB \sim \triangle CEB$.

【解】(2) 旋转前, 点 E 是 BC 的中点, $\therefore BE = \frac{1}{2}BC = 1$.

如图(1), 取 BC 的中点 F , 连接 EF , 则 $BF = \frac{1}{2}BC = 1, \therefore BF = BE$.

由题意得 $\angle CBE = 60^\circ$,
 $\therefore \triangle BEF$ 是等边三角形,
 $\therefore \angle BFE = 60^\circ, EF = BF = CF$, $\therefore \angle BCE = 30^\circ$,
 $\therefore \angle BCE + \angle CBE = 90^\circ$,
 $\therefore \angle BEC = 90^\circ, \therefore CE = \sqrt{BC^2 - BE^2} = \sqrt{3}$.



图(1)

思路分析

(1) 先判断出 $\frac{BC}{AB} = \frac{BE}{BD}$, 再判断出夹角相等, 即可得出结论.

(2) 先判断出 $\triangle BEF$ 是等边三角形, 进而判断出 $\angle BEC = 90^\circ$, 求出 CE , 借助(1)的结论得出比例式, 即可得出结论.

(3) 分两种情况, 先判断出 $\angle AEB = 90^\circ$, 利用勾股定理求出 AE , 进而得出 AD , 最后借助(1)中的结论得出比例式, 即可求出 CE 的长.

由(1)知, $\triangle ADB \sim \triangle CEB$, $\therefore \frac{AD}{CE} = \frac{AB}{BC}$, 即

$$\frac{AD}{\sqrt{3}} = \frac{2\sqrt{2}}{2}, \therefore AD = \sqrt{6}.$$

(3) ①当点 E 在线段 AD 上时, 如图(2).

$\because \angle BED = 90^\circ$,
 $\therefore \angle AEB = 90^\circ$.

在 $\text{Rt} \triangle ABE$ 中, 根据

勾股定理得 $AE = \sqrt{AB^2 - BE^2} = \sqrt{(2\sqrt{2})^2 - 1^2} = \sqrt{7}$. 在 $\text{Rt} \triangle BED$ 中, $DE = BE = 1, \therefore AD = AE + DE = \sqrt{7} + 1$.

由(1)知, $\triangle ADB \sim \triangle CEB$, $\therefore \frac{AD}{CE} = \frac{AB}{BC}$, 即

$$\frac{\sqrt{7}+1}{CE} = \frac{2\sqrt{2}}{2}, \therefore CE = \frac{\sqrt{14}+\sqrt{2}}{2}.$$

②当点 E 在线段 AD 的延长线上时, 如图(3).

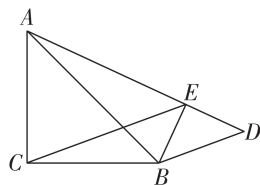
同①的方法得 $AE = \sqrt{7}$,
 $\therefore AD = AE - DE = \sqrt{7} - 1$.

由(1)知 $\triangle ADB \sim \triangle CEB$,

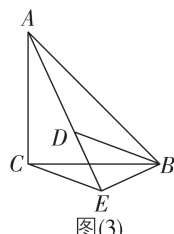
$$\therefore \frac{AD}{CE} = \frac{AB}{BC}, \text{ 即 } \frac{\sqrt{7}-1}{CE} = \frac{2\sqrt{2}}{2},$$

$$\therefore CE = \frac{\sqrt{14}-\sqrt{2}}{2}.$$

综上所述, CE 的长为 $\frac{\sqrt{14}+\sqrt{2}}{2}$ 或 $\frac{\sqrt{14}-\sqrt{2}}{2}$.



图(2)



图(3)

第8章 锐角三角函数

8.1 正切、正弦、余弦

课时1 正切

刷基础

1. C 【解析】设 $AC = b, BC = a. \because \angle C = 90^\circ$,
 $\therefore \tan A = \frac{BC}{AC} = \frac{a}{b}$. $\because$ 各边都扩大 3 倍, $\therefore$ 扩大后 $AC = 3b, BC = 3a, \therefore \tan A = \frac{3a}{3b} = \frac{a}{b}, \therefore \tan A$ 的值不变. 故选 C.

2. C 【解析】设 $AB = 2x, AC = x. \because \angle C = 90^\circ$,
 $\therefore BC = \sqrt{AB^2 - AC^2} = \sqrt{3}x, \therefore \tan B = \frac{AC}{BC} = \frac{x}{\sqrt{3}x} = \frac{\sqrt{3}}{3}$. 故选 C.

3. 【解】(1) $\because$ 四边形 $ABCD$ 是正方形, $AB = 4$,
 $\therefore AB = BC = 4, \angle B = 90^\circ, \therefore AC = \sqrt{AB^2 + BC^2} = \sqrt{4^2 + 4^2} = 4\sqrt{2}$. 故答案为 $4\sqrt{2}$.

(2) $\because$ 四边形 $ABCD$ 是正方形, $\therefore AD = AB = 4$,
 $\angle D = 90^\circ, \angle CAD = \angle ACD = 45^\circ$.

$\because$ 点 A 与点 A' 关于 PQ 对称, $\therefore \triangle APQ$ 与 $\triangle A'PQ$ 关于 PQ 对称, $\therefore \angle DAC = \angle QA'P = \angle QCD = 45^\circ, AP = PA'$.

$\because \angle QA'D = \angle QA'P + \angle PA'D, \angle QA'D = \angle QCD + \angle A'QC, \therefore \angle PA'D = \angle A'QC$.

$\because AD = AB = 4, AP = 3PD, \therefore PD = 1, AP = PA' = 3$.

在 $\text{Rt} \triangle PDA'$ 中, 由勾股定理得 $A'D = 2\sqrt{2}$,

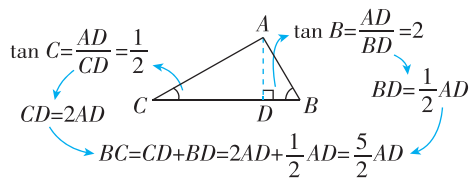
$$\therefore \tan \angle A'QC = \tan \angle PA'D = \frac{PD}{A'D} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}.$$

4. **D** 【解析】 $\because AB = AC, AD \perp BC$ 于点 D ,
 $\therefore \angle DAB = \angle DAC, \therefore \tan \angle DAB = \tan \angle DAC =$
 $\frac{1}{2}, \therefore \tan \angle DAB = \frac{BD}{AD} = \frac{1}{2}$, 即 $\frac{BD}{4} = \frac{1}{2}$, 解得
 $BD = 2, \therefore$ 在 $\text{Rt} \triangle ABD$ 中, $AB = \sqrt{AD^2 + BD^2} =$
 $\sqrt{4^2 + 2^2} = 2\sqrt{5}. \therefore$ 点 E 为 AB 的中点, $\therefore DE =$
 $\frac{1}{2}AB = \frac{1}{2} \times 2\sqrt{5} = \sqrt{5}$. 故选 D.

5. **8** 【解析】 $\because$ 点 A 的坐标为 $(0, \sqrt{3})$, $\therefore OA =$
 $\sqrt{3}$. 在 $\text{Rt} \triangle ABO$ 中, $\angle AOB = 90^\circ, \tan \angle ABO =$
 $\sqrt{3}, \therefore \frac{OA}{OB} = \sqrt{3}, \therefore OB = 1, \therefore AB = \sqrt{OA^2 + OB^2} =$
 $2, \therefore$ 菱形 $ABCD$ 的周长为 $4AB = 8$. 故答案
 为 8.

6. **20**

识图解题



【解析】过点 A 作 $AD \perp BC$ 于点 $D. \therefore \tan B =$
 $2, \therefore \frac{AD}{BD} = 2. \therefore \tan C = \frac{1}{2}, \therefore \frac{AD}{CD} = \frac{1}{2}$. 设 $AD = x$,
 则 $BD = \frac{1}{2}x, DC = 2x$, 故 $\frac{1}{2}x + 2x = 10$, 解得 $x =$
 $4, \therefore$ 这块草地的面积为 $\frac{1}{2} \times AD \times BC = \frac{1}{2} \times 4 \times$
 $10 = 20 (\text{m}^2)$. 故答案为 20.

刷易错

7. **D** 【解析】如图, 取格点 D ,
 连接 BD, CD . 由网格的特点
 得, A, C, D 三点共线. $\therefore BD =$
 $\sqrt{1^2 + 2^2} = \sqrt{5}, AD = \sqrt{2^2 + 4^2} =$
 $2\sqrt{5}, AB = \sqrt{3^2 + 4^2} = 5,$
 $\therefore BD^2 + AD^2 = AB^2, \therefore \triangle ABD$ 为直角三角形,
 $\angle ADB = 90^\circ, \therefore \tan \angle BAD = \frac{BD}{AD} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$. 故
 选 D.

刷提升

1. **C** 【解析】如图, 过点 P 作 $PQ \perp x$ 轴于点 Q .
 $\because OP \parallel AB, \therefore$ 易得 $\triangle OCP \sim \triangle BCA, \therefore CP :$

思路分析

根据 $OP \parallel AB$,
 证明 $\triangle OCP \sim$
 $\triangle BCA$, 得到
 $CP : AC = OC :$
 $BC = 1 : 2$. 过点
 P 作 $PQ \perp x$ 轴
 于点 Q . 根据
 $\angle AOC = \angle AQP =$
 90° , 得到 $CO \parallel$
 PQ , 根据平行
 线分线段成比
 例得到 $OQ :$
 $AO = CP : AC =$
 $1 : 2$, 根据 $P(1,$
 $1)$, 得到 $PQ =$
 $OQ = 1$, 从而得
 到 $AO = 2$, 根据
 正切的定义即
 可得到 $\tan \angle OAP$
 的值.

关键点拨

分三段讨论:
 点 P 从点 M
 移动到点 B ;
 点 P 从点 B
 移动到 BC 中
 点; 点 P 从 BC
 中点移动到
 点 N .

易错警示

根据定义求锐
 角的正切值
 时, 如果题中
 该锐角所在的
 三角形不是直
 角三角形, 一
 定要先构造含
 该锐角的直角
 三角形, 再求
 其正切值.

$$AC = OC : BC = 1 : 2.$$

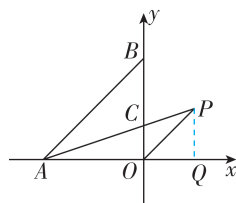
$$\therefore \angle AOC = \angle AQP = 90^\circ,$$

$$\therefore CO \parallel PQ, \therefore OQ : AO =$$

$$CP : AC = 1 : 2. \therefore P(1,$$

$$1), \therefore PQ = OQ = 1,$$

$$\therefore AO = 2, \therefore \tan \angle OAP = \frac{PQ}{AQ} = \frac{1}{3}. \text{ 故选 C.}$$



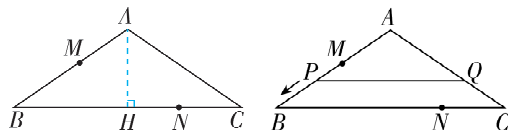
2. **A** 【解析】如图(1), 过点 A 作 $AH \perp BC$ 于点

$$H. \because AB = AC, BC = 8, \therefore BH = CH = \frac{1}{2}BC = 4.$$

$$\therefore \tan C = \frac{AH}{CH} = \frac{3}{4}, \therefore AH = \frac{3}{4}CH = \frac{3}{4} \times 4 = 3,$$

$$\therefore AB = AC = \sqrt{AH^2 + CH^2} = \sqrt{3^2 + 4^2} = 5. \therefore AM =$$

$$CN = 2, \therefore BM = AB - AM = 3, BN = BC - CN = 6.$$

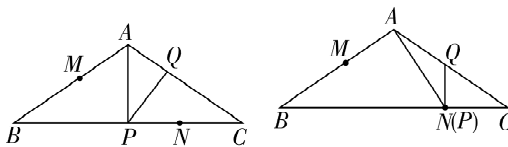


图(1)

图(2)

当点 P 在 MB 上移动时, 如图(2). $\because \angle APQ =$
 $\angle B, \therefore PQ \parallel BC, \therefore$ 易得 $CQ = BP, \therefore$ 当点 P 从
 点 M 移动到点 B 时, 点 Q 的移动路径长 $=$
 $BM = 3$.

当点 P 在 BC 上, 且移动到 BC 中点时, 如
 图(3). $\because AB = AC, BP = CP = 4, \therefore \angle B = \angle C,$
 $AP \perp BC. \therefore \angle APQ = \angle B, \angle APC = \angle B +$
 $\angle BAP = \angle APQ + \angle CPQ, \therefore \angle CPQ = \angle BAP,$
 $\therefore \triangle ABP \sim \triangle PCQ, \therefore \frac{AB}{PC} = \frac{BP}{CQ}, \text{ 即 } \frac{5}{4} = \frac{4}{CQ},$
 $\therefore CQ = \frac{16}{5}.$



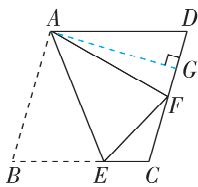
图(3)

图(4)

当点 P 从 BC 中点移动到点 N 时, 如图(4),
 同理可得 $\triangle ABP \sim \triangle PCQ, \therefore \frac{AB}{PC} = \frac{BP}{CQ}, \text{ 即 } \frac{5}{2} =$
 $\frac{6}{CQ}, \therefore CQ = \frac{12}{5}$. 综上, 整个运动过程点 Q 移动
 的路径长为 $3 + \frac{16}{5} + \left(\frac{16}{5} - \frac{12}{5}\right) = 7$. 故选 A.

3. **$\sqrt{15}$** 【解析】如图, 过点 A 作 $AG \perp CD$ 于点
 $G. \because$ 四边形 $ABCD$ 为菱形, 菱形 $ABCD$ 沿 AE
 翻折, 点 B 恰好落在 CD 的中点 F 处, $\therefore AB =$

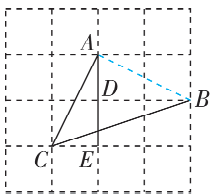
$AD, AB=AF, \angle ABE = \angle D$,
 $\therefore AD=AF, \therefore \triangle ADF$ 为等腰三角形.
 $\because AG \perp DF, \therefore$ 点 G 为 DF 的中点.
 $\because$ 点 F 为 CD 的中点, $\therefore AD=CD=4DG$.
 设 $DG=a$, 则 $AD=4a$. 在 $\text{Rt} \triangle ADG$ 中, 由勾股定理得 $AD^2=AG^2+DG^2, \therefore (4a)^2=AG^2+a^2, \therefore AG=\sqrt{15}a, \therefore \tan \angle ABE = \tan D = \frac{AG}{DG} = \sqrt{15}$. 故答案为 $\sqrt{15}$.



思路分析
 过点 A 作 $AG \perp DF$. 利用翻折的性质和菱形的性质得出 $\triangle ADF$ 为等腰三角形, 由等腰三角形的性质可得点 G 为 DF 中点, 由点 F 为 CD 中点可得 $AD=CD=4DG$, 进一步求解即可.

4. 1 【解析】如图, 连接 AB .

$\because DB \parallel CE, \therefore \angle DBC = \angle BCE$. 在 $\triangle ACE$ 和 $\triangle BAD$ 中, $\begin{cases} AE=BD, \\ \angle AEC = \angle BDA = 90^\circ, \\ CE=AD, \end{cases}$

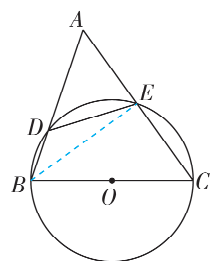


$\therefore \triangle ACE \cong \triangle BAD (SAS), \therefore \angle ABD = \angle CAE, AC=BA$.
 $\because \angle BAD + \angle ABD = 90^\circ, \therefore \angle BAD + \angle CAE = 90^\circ$, 即 $\angle CAB = 90^\circ, \therefore \triangle CAB$ 是等腰直角三角形,
 $\therefore \tan \angle ABC = \frac{AC}{AB} = 1. \therefore \angle ABC = \angle ABD + \angle DBC = \angle CAE + \angle BCE, \therefore \tan (\angle CAE + \angle BCE) = \tan \angle ABC = 1$. 故答案为 1.

关键点拨
 连接 AB , 证出 $\triangle ACE \cong \triangle BAD$, 进而得出 $\triangle CAB$ 是等腰直角三角形是解题的关键.

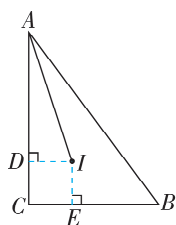
5. $\frac{6}{5}$ 【解析】连接 BE , 如图.

$\because BC$ 是 $\odot O$ 的直径, $\therefore \angle BEC = 90^\circ, \therefore \angle AEB = 90^\circ$.
 $\because$ 在 $\text{Rt} \triangle ABE$ 中, $\tan A = \frac{BE}{AE} = \frac{4}{3}, \therefore$ 设 $BE = 4a, AE = 3a, \therefore AB = \sqrt{AE^2 + BE^2} = \sqrt{(3a)^2 + (4a)^2} = 5a$.
 $\because$ 四边形 $BCED$ 是 $\odot O$ 的内接四边形, $\therefore \angle BDE + \angle C = 180^\circ$.
 $\because \angle BDE + \angle ADE = 180^\circ, \therefore \angle ADE = \angle C$.
 $\because \angle A = \angle A, \therefore \triangle AED \sim \triangle ABC, \therefore \frac{DE}{CB} = \frac{AE}{AB} = \frac{3a}{5a} = \frac{3}{5}$, 即 $\frac{DE}{2} = \frac{3}{5}$, 解得 $DE = \frac{6}{5}$. 故答案为 $\frac{6}{5}$.



6. 【解】如图, 分别过 I 点作 $ID \perp AC$ 于 $D, IE \perp CB$ 于 E .

$\because \angle ACB = 90^\circ, \therefore$ 四边形 $DCEI$ 是矩形.
 $\because I$ 为 $\triangle ABC$ 的内心, $\therefore ID = IE, \therefore$ 四边形 $DCEI$ 是正方形.



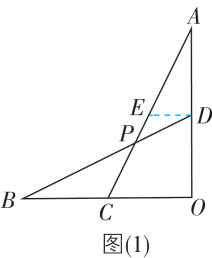
形, $\therefore ID=DC$.

$\because$ 在 $\text{Rt} \triangle ABC$ 中, $BC=3, AB=5, \therefore AC = \sqrt{AB^2 - BC^2} = \sqrt{5^2 - 3^2} = 4, \therefore \triangle ABC$ 的周长为 $3+4+5=12, \therefore S_{\triangle ABC} = \frac{1}{2} \times 12 \times ID = \frac{1}{2} \times 3 \times 4, \therefore ID=1, \therefore ID=DC=1, \therefore AD=4-1=3, \therefore \tan \angle IAC = \frac{ID}{AD} = \frac{1}{3}$.

刷素养

7. (1) 2 (2) $\frac{1}{2}$ (3) $\frac{\sqrt{n}}{n}$

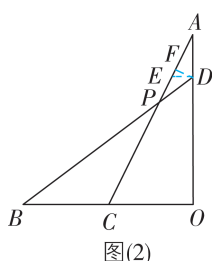
【解析】(1) 如图 (1), 过点 D 作 $DE \parallel CO$ 交 AC 于点 E .
 $\because D$ 为 OA 中点, $\therefore$ 易得 $AE=CE = \frac{1}{2}AC, \frac{DE}{CO} = \frac{1}{2}$.



$\because C$ 为 OB 中点, $\therefore BC=CO, \therefore \frac{DE}{BC} = \frac{1}{2}$.
 $\because DE \parallel CO, \therefore \angle EDP = \angle PBC. \therefore \angle EPD = \angle CPB, \therefore \triangle EDP \sim \triangle CBP, \therefore \frac{PE}{PC} = \frac{DE}{BC} = \frac{1}{2}, \therefore PC = \frac{2}{3}CE = \frac{1}{3}AC, \therefore \frac{AP}{PC} = \frac{AC-PC}{PC} = \frac{\frac{2}{3}AC}{\frac{1}{3}AC} = 2$.

2. 故答案为 2.

(2) 如图 (2), 过点 D 作 $DE \parallel BO$ 交 AC 于点 E , 则 $\triangle AED \sim \triangle ACO, \triangle DEP \sim \triangle BCP. \therefore \frac{AD}{AO} = \frac{1}{4}, \therefore \frac{DE}{CO} = \frac{AE}{AC} = \frac{1}{4}, \therefore CE = \frac{3}{4}AC$.



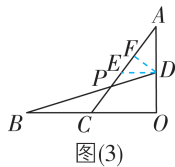
$\because C$ 为 OB 中点, $\therefore BC=CO, \therefore \frac{DE}{BC} = \frac{1}{4}, \therefore \frac{PE}{PC} = \frac{DE}{BC} = \frac{1}{4}, \therefore PC = \frac{4}{5}CE = \frac{3}{5}AC$.

过点 D 作 $DF \perp AC$, 垂足为 F . 设 $AD=a$, 则 $AO=4a. \because OA=OB, C$ 为 OB 中点, $\therefore CO=2a$.
 在 $\text{Rt} \triangle ACO$ 中, $AC = \sqrt{AO^2 + CO^2} = \sqrt{(4a)^2 + (2a)^2} = 2\sqrt{5}a$. 又 $\because \angle AFD = \angle AOC = 90^\circ, \angle A = \angle A, \therefore \triangle ADF \sim \triangle ACO, \therefore \frac{AF}{AO} = \frac{DF}{CO} = \frac{AD}{AC} = \frac{a}{2\sqrt{5}a}, \therefore AF = \frac{2\sqrt{5}}{5}a, DF =$

$$\frac{\sqrt{5}}{5}a, \therefore PF = AC - AF - PC = 2\sqrt{5}a - \frac{2\sqrt{5}}{5}a - \frac{3}{5} \times 2\sqrt{5}a = \frac{2\sqrt{5}}{5}a, \therefore \tan \angle BPC = \tan \angle FPD = \frac{DF}{PF} = \frac{1}{2}.$$

故答案为 $\frac{1}{2}$.

(3) 如图(3), 与(2)的方法相同, 作 $DE \parallel OB$ 交 AC 于 E , 过点 D 作 $DF \perp AC$, 垂足为 F . 设 $AD = b$, 则可得 $DF = \frac{1}{\sqrt{n+1}}b$,



$$PF = \frac{\sqrt{n}}{\sqrt{n+1}}b, \therefore \tan \angle BPC = \tan \angle FPD = \frac{DF}{PF} = \frac{\sqrt{n}}{n}.$$

故答案为 $\frac{\sqrt{n}}{n}$.

课时2 正弦、余弦

刷基础

1. C 【解析】在 $\text{Rt} \triangle ABC$ 中, 由勾股定理, 得

$$AB = \sqrt{BC^2 + AC^2} = \sqrt{1^2 + 2^2} = \sqrt{5}, \therefore \cos B = \frac{BC}{AB} = \frac{1}{\sqrt{5}}.$$

$$\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}. \text{ 故选 C.}$$

2. C 【解析】

A	在 $\text{Rt} \triangle ABC$ 中, $\sin B = \frac{AC}{CB}$; 在 $\text{Rt} \triangle ABD$ 中, $\sin B = \frac{AD}{AB}$, $\therefore \sin B \neq \frac{AC}{AB}$, 故 A 错误
	在 $\text{Rt} \triangle ABC$ 中, $\cos C = \frac{AC}{CB}$; 在 $\text{Rt} \triangle ACD$ 中, $\cos C = \frac{CD}{AC}$, $\therefore \cos C \neq \frac{AD}{CD}$, 故 B 错误
C	在 $\text{Rt} \triangle ABC$ 中, $\sin C = \frac{AB}{BC}$, 故 C 正确
D	在 $\text{Rt} \triangle ABC$ 中, $\tan C = \frac{AB}{AC}$; 在 $\text{Rt} \triangle ACD$ 中, $\tan C = \frac{AD}{CD}$, $\therefore \tan C \neq \frac{AD}{BD}$, 故 D 错误

刷有所得

若在 $\text{Rt} \triangle ABC$ 中, $\angle C = 90^\circ$, 则 $\sin A$ 等于对边比斜边, $\cos A$ 等于邻边比斜边, $\tan A$ 等于对边比邻边.

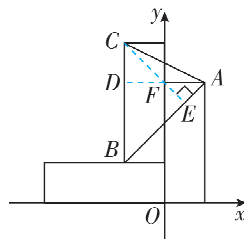
关键点拨

作辅助线, 将 $\angle BAC$ 放在直角三角形中是解题的关键.

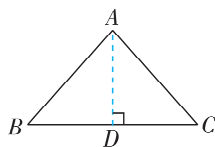
3. $\frac{3\sqrt{10}}{10}$ 【解析】如图, 过 C 作 $CE \perp AB$ 于 E , 延长 AF 交 BC 于 D . 依题意得 $BC = 3, AD = BD = 2, CD = 1, AD \perp BC$, $\therefore$ 在 $\text{Rt} \triangle ADC$ 中, $AC = \sqrt{AD^2 + CD^2} = \sqrt{5}$, 在 $\text{Rt} \triangle ADB$ 中, $AB =$

$$\sqrt{AD^2 + BD^2} = 2\sqrt{2}. \therefore S_{\triangle ABC} = \frac{1}{2} AB \cdot CE = \frac{1}{2} BC \cdot AD, \therefore CE = \frac{BC \cdot AD}{AB} = \frac{3\sqrt{2}}{2},$$

$$\therefore \sin \angle BAC = \frac{CE}{AC} = \frac{3\sqrt{10}}{10}.$$



4. C 【解析】如图, 过点 A 作 $AD \perp BC$ 于点 D . $\therefore AB = AC = 6, \therefore BC = 2BD. \therefore \cos B = \frac{BD}{AB} = \frac{2}{3}, \therefore BD = 4, \therefore BC = 8$. 故选 C.



5. $\frac{4\sqrt{5}}{3}$ 【解析】过点 O 作 $OD \perp AB$ 于 $D. \therefore OA = 2, \sin A = \frac{OD}{AO} = \frac{2}{3}, \therefore OD = \frac{4}{3}, \therefore AD = \sqrt{2^2 - \left(\frac{4}{3}\right)^2} = \frac{2\sqrt{5}}{3}, \therefore AB = 2AD = \frac{4\sqrt{5}}{3}$. 故答案为 $\frac{4\sqrt{5}}{3}$.

6. 【解】(1) $\because AD = BD, \therefore \angle A = \angle ABD$. 又 $\angle BDC = \angle A + \angle ABD = 30^\circ, \therefore \angle A = 15^\circ. \therefore \angle BDC = 30^\circ, BC = a, \therefore BD = 2a, \therefore CD = \sqrt{BD^2 - BC^2} = \sqrt{3}a. \therefore AD = BD, AC = AD + CD, \therefore AC = BD + CD = (2 + \sqrt{3})a, \therefore \tan A = \frac{BC}{AC} = \frac{a}{(2 + \sqrt{3})a} = 2 - \sqrt{3}.$

(2) 在 $\text{Rt} \triangle ABC$ 中, $AC = (2 + \sqrt{3})a, BC = a, \therefore AB = \sqrt{AC^2 + BC^2} = \sqrt{8 + 4\sqrt{3}}a = \sqrt{6 + 4\sqrt{3} + 2}a = \sqrt{(\sqrt{6})^2 + 2 \times \sqrt{6} \times \sqrt{2} + (\sqrt{2})^2}a = (\sqrt{6} + \sqrt{2})a, \therefore \sin A = \frac{BC}{AB} = \frac{a}{(\sqrt{6} + \sqrt{2})a} = \frac{\sqrt{6} - \sqrt{2}}{4}.$

$$\therefore DE \perp AB, \therefore \sin A = \frac{DE}{AD} = \frac{DE}{2a} = \frac{\sqrt{6} - \sqrt{2}}{4},$$

$$\therefore DE = \frac{\sqrt{6} - \sqrt{2}}{2}a.$$

7. ①②③④ 【解析】 $\because \angle BAC = 90^\circ, AD \perp BC, \therefore \angle \alpha + \angle \beta = 90^\circ, \angle B + \angle \beta = 90^\circ, \angle B + \angle C =$

90° , $\therefore \angle \alpha = \angle B$, $\angle \beta = \angle C$, $\therefore \sin \alpha = \sin B$, 故①正确; $\sin \beta = \sin C$, 故②正确; $\therefore$ 在 $\text{Rt}\triangle ABC$ 中, $\sin B = \frac{AC}{BC}$, $\cos C = \frac{AC}{BC}$, $\therefore \sin B = \cos C$, 故③正确; $\therefore \sin \alpha = \sin B$, $\cos \beta = \cos C$, $\sin B = \cos C$, $\therefore \sin \alpha = \cos \beta$, 故④正确. 故答案为①②③④.

8. 40° 【解析】 $\therefore \sin(80^\circ - \alpha) = \cos 50^\circ$, $\therefore 80^\circ - \alpha = 90^\circ - 50^\circ$, $\therefore \alpha = 40^\circ$.

刷易错

9. $\frac{3}{5}, \frac{4}{5}$ 或 $\frac{\sqrt{7}}{4}, \frac{3}{4}$ 【解析】设两个锐角分别是 α 和 β ($\alpha < \beta$). 当直角边长是 6, 8 时, 斜边长是 10, 则 $\sin \alpha = \frac{3}{5}$, $\sin \beta = \frac{4}{5}$; 当斜边长是 8 时, 另一条直角边长是 $2\sqrt{7}$, 则 $\sin \alpha = \frac{\sqrt{7}}{4}$, $\sin \beta = \frac{3}{4}$.

刷提升

1. A 【解析】如图, $\angle C = 90^\circ$, 则 $\tan A = \frac{a}{b}$, $\sin A = \frac{a}{c}$. $\therefore b < c$, $\therefore \tan A > \sin A$, 故 A 选项正确. $\therefore 0 < \sin A < 1$, $\therefore$ 易得 $\sqrt{(\sin A - 1)^2} = 1 - \sin A$, 故 B 选项错误. $\therefore \cos A = \frac{b}{c}$, $\tan(90^\circ - \angle A) = \tan B = \frac{b}{a}$, $\therefore \cos A \neq \tan(90^\circ - \angle A)$, 故 C 选项错误. $\therefore \sin A = \frac{a}{c}$, $\cos A = \frac{b}{c}$, $\therefore \sin A + \cos A = \frac{a+b}{c} \neq 1$, 故 D 选项错误. 故选 A.

2. B 【解析】如图, 连接 BF . $\therefore CE$ 是斜边 AB 上的中线, $EF \perp AB$, $\therefore EF$ 是 AB 的垂直平分线, $\therefore FB = FA$, $BE = AE$, $\therefore S_{\triangle AFE} = S_{\triangle BFE} = 25$, $\angle FBA = \angle A$, $\therefore S_{\triangle AFB} = 50 = \frac{1}{2} AF \cdot BC$. $\therefore CE = AE = BE = \frac{1}{2} AB$, $\therefore \angle A = \angle FBA = \angle ACE$. 又 $\therefore \angle BCA = 90^\circ = \angle BEF$, $\therefore \angle CBF = 90^\circ - \angle BFC = 90^\circ - 2\angle A$, $\angle CEF = 90^\circ - \angle BEC = 90^\circ - 2\angle A$, $\therefore \angle CEF = \angle FBC$, $\therefore \sin \angle CEF = \sin \angle FBC = \frac{3}{5} = \frac{CF}{BF}$. 设 $BF = AF = x$, $\therefore CF = \frac{3}{5}x$. $\therefore \frac{1}{2} BC \cdot AF = 50$, $\therefore BC = \frac{100}{x}$.

关键点拨

连接 CD , 根据角平分线的定义和同弧所对的角相等推出 $\triangle CDN \sim \triangle ABN$ 是解题的关键.

易错警示

对于直角三角形, 如果没有指明所给边长是直角边长还是斜边长, 解题时应注意分类讨论.

$\therefore BC^2 + CF^2 = BF^2$, $\therefore \left(\frac{100}{x}\right)^2 + \left(\frac{3}{5}x\right)^2 = x^2$, $\therefore x = 5\sqrt{5}$ (负值已舍去), $\therefore BC = \frac{100}{x} = 4\sqrt{5}$. 故选 B.

3. $\frac{2\sqrt{5}}{5}$ 【解析】连接 CD . $\therefore AD$ 平分 $\angle CAB$, $\therefore \angle CAD = \angle BAD$. $\therefore \widehat{BD} = \widehat{BD}$, $\therefore \angle BCD = \angle BAD$. 同理, $\angle D = \angle B$, $\therefore \triangle CDN \sim \triangle ABN$, $\therefore \frac{DN}{BN} = \frac{CN}{AN} = \frac{\sqrt{5}}{5}$. 设 $CN = x$, 则 $AN = \sqrt{5}x$. $\therefore AB$ 是圆 O 的直径, $\therefore \angle ACB = 90^\circ$, $\therefore$ 在 $\text{Rt}\triangle ACN$ 中, $AC = \sqrt{AN^2 - CN^2} = \sqrt{(\sqrt{5}x)^2 - x^2} = 2x$, $\therefore \cos \angle BAD = \cos \angle CAD = \frac{AC}{AN} = \frac{2x}{\sqrt{5}x} = \frac{2\sqrt{5}}{5}$. 故答案为 $\frac{2\sqrt{5}}{5}$.

4. 【解】(1) 如图, 过点 A 作 $AD \perp BC$ 于点 D . $\therefore AB = AC = 10$, $\therefore BC = 2BD$. 在 $\text{Rt}\triangle ABD$ 中, $\therefore \sin \angle ABC = \frac{AD}{AB} = \frac{3}{5}$, $\therefore AD = AB \cdot \sin \angle ABC = 10 \times \frac{3}{5} = 6$, $\therefore BD = \sqrt{AB^2 - AD^2} = 8$, 则 $BC = 2BD = 16$. (2) 如图, 过点 B' 作 $B'E \perp BC$ 于点 E , 连接 BB' . 根据题意知 $B'C = BC = 16$, $\angle ABC = \angle ACB = \angle A'CB'$, $\therefore \sin \angle BCB' = \sin \angle ABC = \frac{3}{5} = \frac{B'E}{B'C}$, $\therefore B'E = 16 \times \frac{3}{5} = \frac{48}{5}$, $\therefore CE = \sqrt{B'C^2 - B'E^2} = \frac{64}{5}$, $\therefore BE = BC - CE = 16 - \frac{64}{5} = \frac{16}{5}$, $\therefore BB' = \sqrt{BE^2 + B'E^2} = \sqrt{\left(\frac{16}{5}\right)^2 + \left(\frac{48}{5}\right)^2} = \frac{16\sqrt{10}}{5}$, $\therefore$ 点 B 和点 B' 之间的距离等于 $\frac{16\sqrt{10}}{5}$.

刷素养

5. 【解】(1) $\therefore 270^\circ < \alpha < 360^\circ$, $\therefore x > 0, y < 0$, $\therefore \angle \alpha$ 的三角函数值 $\sin \alpha, \cos \alpha, \tan \alpha$, 其中取正值的是 $\cos \alpha$. 故答案为 $\cos \alpha$. (2) $\therefore \angle \alpha$ 的终边与直线 $y = 2x$ 重合, $\therefore$ 易得 $\sin \alpha = \frac{2\sqrt{5}}{5}$, $\cos \alpha = \frac{\sqrt{5}}{5}$ 或 $\sin \alpha = -\frac{2\sqrt{5}}{5}$,

$$\cos \alpha = -\frac{\sqrt{5}}{5}, \therefore \sin \alpha + \cos \alpha = \frac{3\sqrt{5}}{5} \text{ 或 } \sin \alpha + \cos \alpha = -\frac{3\sqrt{5}}{5}.$$

$$(3) \text{ 由题意得, } \cos \alpha = \frac{x}{r} = \frac{\sqrt{2}}{4}x, \therefore r = 2\sqrt{2}.$$

$$\therefore r = \sqrt{x^2 + (\sqrt{5})^2}, \therefore x^2 = 3. \therefore \angle \alpha \text{ 是钝角,}$$

$$\therefore x = -\sqrt{3}, \therefore \tan \alpha = \frac{y}{x} = \frac{\sqrt{5}}{-\sqrt{3}} = -\frac{\sqrt{15}}{3}.$$

8.2 锐角三角函数的计算

课时 1 特殊角的三角函数值

刷基础

1. D 【解析】

A $\sin^2 60^\circ + \sin^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{4} = 1$, 此选项不符合题意

B $\tan 45^\circ = 1, \tan 30^\circ = \frac{\sqrt{3}}{3}$, 所以 $\tan 45^\circ > \tan 30^\circ$, 此选项不符合题意

C $\tan 45^\circ = 1, \sin 45^\circ = \frac{\sqrt{2}}{2}$, 所以 $\tan 45^\circ > \sin 45^\circ$, 此选项不符合题意

D $\sin 30^\circ + \cos 30^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2} \neq 1$, 此选项符合题意

2. D 【解析】如图, 连接 BC . 由题意可得, $OB = OC = BC$, 则 $\triangle OBC$ 是等边三角形,

$$\therefore \sin \angle AOC = \sin 60^\circ = \frac{\sqrt{3}}{2}. \text{ 故}$$

选 D.

3. $\sqrt{3}$ 【解析】 $3\tan 45^\circ = 3 \times 1 = 3, 2\sin 60^\circ = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}, 4\cos 60^\circ = 4 \times \frac{1}{2} = 2. \therefore 3 > 2 > \sqrt{3},$
 $\therefore \min\{3\tan 45^\circ, 2\sin 60^\circ, 4\cos 60^\circ\} = \min\{3, \sqrt{3}, 2\} = \sqrt{3}.$ 故答案为 $\sqrt{3}$.

4. 【解】(1) 原式 $= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 + \sqrt{2} \times \frac{\sqrt{3}}{2} \times 1 =$

$$\frac{1}{4} + \frac{1}{2} + \frac{\sqrt{6}}{2} = \frac{3}{4} + \frac{\sqrt{6}}{2} = \frac{3+2\sqrt{6}}{4}.$$

关键点拨

推出 $30^\circ < \alpha < 60^\circ$ 是解题的关键.

关键点拨

熟记 $30^\circ, 45^\circ, 60^\circ$ 角的三角函数值是解题的关键.

易错警示

本题容易因误认为三角函数值的和差等于对应角度的和差的三角函数值而出错.

$$(2) \text{ 原式} = \sqrt{\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right)^2} - \left|\sqrt{3} - \frac{\sqrt{3}}{2}\right| = \frac{\sqrt{2}}{2} - \frac{1}{2} - \left(\sqrt{3} - \frac{\sqrt{3}}{2}\right) = \frac{\sqrt{2}}{2} - \frac{1}{2} - \sqrt{3} + \frac{\sqrt{3}}{2} = \frac{\sqrt{2}-1-\sqrt{3}}{2}.$$

5. 等边 【解析】 $\therefore \left|\sin A - \frac{\sqrt{3}}{2}\right| + \left(\frac{1}{2} - \cos B\right)^2 =$

$$0, \angle A, \angle B \text{ 均为锐角, } \therefore \sin A = \frac{\sqrt{3}}{2}, \cos B =$$

$$\frac{1}{2}, \therefore \angle A = 60^\circ, \angle B = 60^\circ, \therefore \angle C = 60^\circ,$$

$\therefore \angle A = \angle B = \angle C, \therefore \triangle ABC$ 是等边三角形. 故答案为等边.

6. 45 (答案不唯一) 【解析】 $\therefore \cos 60^\circ =$

$$\frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \frac{1}{2} < \cos \alpha < \frac{\sqrt{3}}{2}, \text{ 且 } \alpha \text{ 为锐角,}$$

$\therefore 30^\circ < \alpha < 60^\circ, \therefore \alpha$ 可取 45° . 故答案为 45 (答案不唯一).

7. (1) 17° (2) 30° 【解析】(1) $\therefore 2\sin(A +$

$$13^\circ) = 1, \therefore \sin(A + 13^\circ) = \frac{1}{2}. \therefore \angle A \text{ 为锐角,}$$

$\therefore \angle A + 13^\circ = 30^\circ, \therefore \angle A = 17^\circ, \therefore$ 锐角 A 的度数为 17° .

$$(2) \therefore 3\tan \alpha - \sqrt{3} = 0, \therefore \tan \alpha = \frac{\sqrt{3}}{3}. \therefore \alpha \text{ 为锐角,}$$

$\therefore \alpha = 30^\circ, \therefore$ 锐角 α 的度数为 30° .

8. 【解】小明的解答过程不正确, 正确解答过程

如下: 在 $\text{Rt} \triangle ABC$ 中, $\therefore \sin A = \frac{BC}{AB} = \frac{\sqrt{3}}{2},$

$$\therefore \angle A = 60^\circ, \therefore \sin \frac{A}{2} = \sin 30^\circ = \frac{1}{2}.$$

刷易错

9. D 【解析】

A $\sin 30^\circ + \sin 30^\circ = \frac{1}{2} + \frac{1}{2} = 1, \sin 60^\circ = \frac{\sqrt{3}}{2},$

故 A 选项错误

B $\tan 60^\circ - \tan 30^\circ = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}, \tan 30^\circ =$

$$\frac{\sqrt{3}}{3}, \text{ 故 B 选项错误}$$

C $\cos(60^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ -$

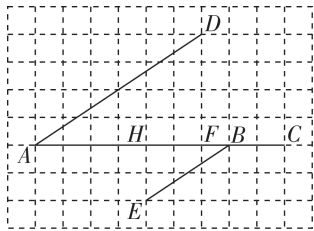
$$\cos 30^\circ = \frac{1}{2} - \frac{\sqrt{3}}{2}, \text{ 故 C 选项错误}$$

D $3\tan 30^\circ = 3 \times \frac{\sqrt{3}}{3} = \sqrt{3}, \text{ 故 D 选项正确}$

刷提升

1. B 【解析】 $\because \triangle AOB$ 绕点 O 逆时针旋转 40° 后得到 $\triangle DOE$, 点 A 恰好落在边 DE 上, $\therefore \angle AOD = 40^\circ$, $\angle OAB = \angle D$, $OA = OD$, $\therefore \angle OAD = \angle D$, $\therefore \angle D = \frac{1}{2}(180^\circ - \angle AOD) = \frac{1}{2} \times (180^\circ - 40^\circ) = 70^\circ$, $\therefore \angle OAB = 70^\circ$. $\because \angle AOB = \angle BOD - \angle AOD = 105^\circ - 40^\circ = 65^\circ$, $\therefore \angle B = 180^\circ - 70^\circ - 65^\circ = 45^\circ$, $\therefore \cos B = \cos 45^\circ = \frac{\sqrt{2}}{2}$. 故选 B.

2. C 【解析】如图所示, $\because$ 小正方形的边长为 1, $\therefore AF = 6$, $DF = 4$, $BH = 3$, $EH = 2$, $\angle AFD = \angle BHE = 90^\circ$. 在 $\text{Rt}\triangle ADF$ 中, $\tan \angle DAC = \frac{DF}{AF} = \frac{4}{6} = \frac{2}{3} \neq \frac{\sqrt{3}}{3}$, 在 $\text{Rt}\triangle BHE$ 中, $\tan \angle EBA = \frac{EH}{BH} = \frac{2}{3} \neq \frac{\sqrt{3}}{3}$. 又 $\because \angle DAC$ 和 $\angle EBA$ 都是锐角, $\tan \angle DAC = \tan \angle EBA$, $\therefore \angle DAC = \angle EBA \neq 30^\circ$, $\therefore \angle DAC + \angle EBA \neq 60^\circ$, 故选 C.



3. 120° 【解析】 $\because \angle A = 90^\circ$, CD 平分 $\angle ACB$, $DE \perp BC$, $\therefore DA = DE$, $\angle DEC = 90^\circ$. $\because DC = DC$, $\therefore \text{Rt}\triangle ADC \cong \text{Rt}\triangle EDC$ (HL), $\therefore AC = EC$. $\because BE = EC$, $\therefore AC = EC = BE = \frac{1}{2}BC$, $\therefore \sin \angle ABC = \frac{AC}{BC} = \frac{1}{2}$, $\therefore \angle ABC = 30^\circ$, $\therefore \angle BDE = 60^\circ$, $\therefore \angle ADE = 120^\circ$, 故答案为 120° .

4. $\sqrt{3}$ 【解析】由作图知 BP 平分 $\angle ABC$, $BA = BE$. $\because$ 在 $\square ABCD$ 中, $AD \parallel BC$, $\angle D = 60^\circ$, $\therefore \angle ABC = 60^\circ$, $\therefore \triangle ABE$ 是等边三角形, $\angle BAD = 120^\circ$. $\because BP$ 平分 $\angle ABC$, $\therefore BO \perp AE$, $AO = OE$, $\angle ABF = \angle EBF = \frac{1}{2} \angle ABC = 30^\circ$. $\because AD \parallel BC$, $\therefore \angle AFB = \angle EBF = 30^\circ$, $\therefore \angle AFB = \angle ABF = 30^\circ$, $\therefore AB = AF$. $\because BO \perp AE$, $\therefore \angle BAO = \angle FAO = \frac{1}{2} \times 120^\circ = 60^\circ$, $\therefore \frac{OF}{OE} =$

思路分析

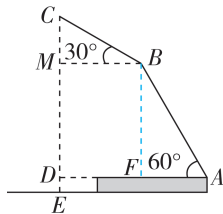
根据 $\sin 30^\circ = \frac{CM}{BC}$, 求出 CM 的长, 根据 $\sin 60^\circ = \frac{BF}{AB}$, 求出 BF 的长, 即可得出 MD 的长, 进而可求出 CE 的长.

关键点拨

(3) 在直角三角形中作辅助线构造含 $2\angle A$ 的直角三角形是解题的关键.

$\frac{OF}{AO} = \tan \angle FAO = \tan 60^\circ = \sqrt{3}$, 故答案为 $\sqrt{3}$.

5. $(17+20\sqrt{3})$ 【解析】如图, 过点 B 作 $BF \perp AD$ 于点 F , 则四边形 $MDFB$ 是矩形, $\therefore MD = BF$. 在 $\text{Rt}\triangle CMB$ 中, $BC =$



30 cm , $\angle CBM = 30^\circ$, $\therefore CM = BC \sin 30^\circ = 30 \times \frac{1}{2} = 15 \text{ (cm)}$. 在 $\text{Rt}\triangle AFB$ 中, $AB = 40 \text{ cm}$, $\angle BAF = 60^\circ$, $\therefore BF = AB \sin 60^\circ = 40 \times \frac{\sqrt{3}}{2} = 20\sqrt{3} \text{ (cm)}$, $\therefore MD = BF = 20\sqrt{3} \text{ cm}$, $\therefore CD = CM + DM = (15 + 20\sqrt{3}) \text{ cm}$, $\therefore CE = CD + DE = (17 + 20\sqrt{3}) \text{ cm}$, 即灯罩顶端 C 到桌面的高度 CE 是 $(17 + 20\sqrt{3}) \text{ cm}$, 故答案为 $(17 + 20\sqrt{3})$.

刷素养

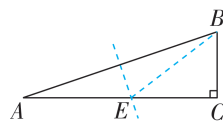
6. 【解】 (1) $\tan 60^\circ = \sqrt{3}$, $\tan 30^\circ = \frac{\sqrt{3}}{3}$, $2 \tan 30^\circ =$

$\frac{2\sqrt{3}}{3}$, $\therefore \tan A \neq 2 \tan \frac{1}{2}A$, 故答案为 $\neq$.

(2) 延长 CA 到点 D , 使 $DA = AB$, 连接 BD , 则 $\angle D = \angle ABD$, $\therefore \angle BAC = \angle D + \angle ABD = 2\angle D$, $\therefore \angle D = \frac{1}{2} \angle BAC$.

在 $\text{Rt}\triangle ABC$ 中, $\angle C = 90^\circ$, $AC = 4$, $BC = 3$, $\therefore AB = \sqrt{AC^2 + BC^2} = 5$, $\therefore AD = AB = 5$, $\therefore CD = AD + AC = 9$, $\therefore \tan \frac{1}{2} \angle BAC = \tan D = \frac{BC}{CD} = \frac{3}{9} = \frac{1}{3}$.

(3) 如图, 作 AB 的垂直平分线交 AC 于点 E , 连接 BE , 则 $AE = BE$, $\therefore \angle A = \angle ABE$, $\therefore \angle BEC = \angle A + \angle ABE = 2\angle A$.

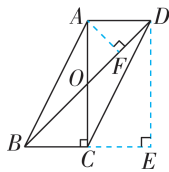


在 $\text{Rt}\triangle ABC$ 中, $\angle C = 90^\circ$, $AC = 3$, $\tan A = \frac{BC}{AC} = \frac{1}{3}$, $\therefore BC = 1$.

设 $AE = BE = x$, 则 $EC = 3 - x$. 在 $\text{Rt}\triangle EBC$ 中, $BE^2 = EC^2 + BC^2$, 即 $x^2 = (3 - x)^2 + 1$, 解得 $x = \frac{5}{3}$, $\therefore AE = BE = \frac{5}{3}$, $EC = \frac{4}{3}$, $\therefore \tan 2A = \tan \angle BEC = \frac{BC}{EC} = \frac{3}{4}$.

$$\tan \angle EBF = \frac{EF}{BF} = \frac{\frac{\sqrt{2}}{2}a}{\sqrt{2}a + \frac{\sqrt{2}}{2}a} = \frac{1}{3}, \text{ 即 } \tan \angle EBC = \frac{1}{3}.$$

4. $\frac{1}{3}$ 【解析】如图,过点 A 作 $AF \perp BD$ 于点 F,过点 D 作 $DE \perp BC$,交 BC 的延长线于点 E,易知四边形 ACED 是矩形, $\therefore CE=AD$. $\because$ 四边形 ABCD 是平行四边形, $\therefore AD=BC$, $OA=OC$. 设 $BC=a$,则 $AD=CE=a$, $\therefore BE=2a$. $\because OA=BC$, $\therefore DE=AC=2a$, $\therefore DE=BE$, $\therefore \triangle BDE$ 是等腰直角三角形, $\therefore BD=2\sqrt{2}a$, $\angle DBE=45^\circ$. $\because AD \parallel BC$, $\therefore \angle ADF=45^\circ$, $\therefore \triangle ADF$ 是等腰直角三角形, $\therefore AF=DF=\frac{\sqrt{2}}{2}a$, $\therefore BF=BD-DF=\frac{3\sqrt{2}}{2}a$, $\therefore \tan \angle ABD=\frac{AF}{BF}=\frac{1}{3}$. 故答案为 $\frac{1}{3}$.



思路分析

过点 A 作 $AF \perp BD$ 于点 F,过点 D 作 $DE \perp BC$,交 BC 的延长线于点 E. 设 $BC=a$,则 $AD=CE=a$,进而得出 $DE=AC=2a$,则 $\triangle BDE$ 是等腰直角三角形,求得 AF, BF 的长,进而根据正切的定义求解.

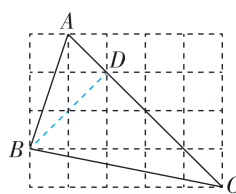
思路分析

通过割补法求出 $\triangle ABC$ 的面积,用勾股定理求出 AB, AC 的长度. 过点 C 作 $CM \perp AB$ 于 M. 利用等面积法求出 AB 边上的高 CM,最后根据正弦定义计算 $\sin A$.

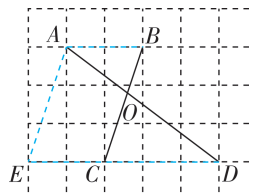
关键点拨

取格点 D,连接 BD,根据勾股定理的逆定理证明 $\triangle ABD$ 是直角三角形,得到 $\angle ADB = \angle BDC = 90^\circ$ 是解本题的关键.

$$\frac{2\sqrt{2}}{\sqrt{26}} = \frac{2\sqrt{13}}{13}. \text{ 故答案为 } \frac{2\sqrt{13}}{13}.$$



(第 1 题图)



(第 2 题图)

2. 3 【解析】如图,连接 AE, AB, ED. $\because AB \parallel EC$, $AB=EC=2$, $\therefore$ 四边形 AECB 是平行四边形, $\therefore AE \parallel BC$. $\because AD=\sqrt{3^2+4^2}=5$, $DE=5$, $\therefore AD=DE=5$, $\therefore \angle DAE = \angle DEA$. $\because AE \parallel BC$, $\therefore \angle DAE = \angle DOC$, $\therefore \angle DOC = \angle DEA$, $\therefore \tan \angle COD = \tan \angle DEA = \frac{3}{1} = 3$,故答案为 3.

3. $\frac{17\sqrt{26}}{130}$ 【解析】设每个小方格的边长均为 1,则 $S_{\triangle ABC} = 4 \times 5 - \frac{1}{2} \times 3 \times 4 - \frac{1}{2} \times 5 \times 1 - \frac{1}{2} \times 3 \times 2 = 8.5$. 由勾股定理得 $AB = \sqrt{5^2+1^2} = \sqrt{26}$, $AC = \sqrt{4^2+3^2} = 5$. 过点 C 作 $CM \perp AB$ 于 M,则 $S_{\triangle ABC} = \frac{1}{2} AB \cdot CM$, $\therefore 8.5 = \frac{1}{2} \times \sqrt{26} \times CM$, $\therefore CM = \frac{17\sqrt{26}}{26}$, $\therefore$ 在 $\text{Rt} \triangle AMC$ 中, $\sin A = \frac{CM}{AC} = \frac{\frac{17\sqrt{26}}{26}}{5} = \frac{17\sqrt{26}}{130}$. 故答案为 $\frac{17\sqrt{26}}{130}$.

8.3 解直角三角形

8.3.1 解直角三角形



刷基础

1. C 【解析】 $\because$ 在 $\text{Rt} \triangle ABC$ 中, $\angle ACB = 90^\circ$,斜边上的中线 $CD=6$, $\therefore AB=12$. $\therefore \sin A = \frac{BC}{AB} = \frac{1}{3}$, $\therefore BC=4$, $\therefore AC = \sqrt{AB^2 - BC^2} = 8\sqrt{2}$, $\therefore S_{\triangle ABC} = \frac{1}{2} AC \cdot BC = 16\sqrt{2}$. 故选 C.
2. A 【解析】 $\because \angle B=45^\circ$, $AD \perp BC$, $\therefore \triangle ABD$ 是等腰直角三角形, $\therefore AD = \frac{\sqrt{2}}{2} AB = \frac{\sqrt{2}}{2} \times 2\sqrt{6} = 2\sqrt{3}$. $\therefore \sin C = \frac{AD}{AC} = \frac{\sqrt{3}}{2}$, $\therefore AC=4$. $\because E, F$ 分别为 AB, BC 的中点, $\therefore EF$ 是 $\triangle ABC$ 的中位线,

大招解读 | 巧设参数法

锐角三角函数的实质就是直角三角形中对应两边长度的比值,所以在解题中有时需要将三角函数转化为线段比,通过设定一个参数,并用含该参数的代数式表示出直角三角形中各边的长,再结合题中条件解决问题.

5. $\frac{12}{13}$ 【解析】 $\because$ 在 $\triangle ABC$ 中, $\angle C=90^\circ$, $\tan A = \frac{5}{12}$, $\therefore$ 设 $AC=12k$, $BC=5k$, 则 $AB = \sqrt{(12k)^2 + (5k)^2} = 13k$, $\therefore \sin B = \frac{AC}{AB} = \frac{12k}{13k} = \frac{12}{13}$. 故答案为 $\frac{12}{13}$.



微专题

1. $\frac{2\sqrt{13}}{13}$ 【解析】如图,取格点 D,连接 BD. 由题意得 $AB = \sqrt{1^2+3^2} = \sqrt{10}$, $BC = \sqrt{1^2+5^2} = \sqrt{26}$, $BD = \sqrt{2^2+2^2} = 2\sqrt{2}$, $AD = \sqrt{1^2+1^2} = \sqrt{2}$. $\therefore AD^2 + BD^2 = 10$, $AB^2 = 10$, $\therefore AD^2 + BD^2 = AB^2$, $\therefore \triangle ABD$ 是直角三角形,且 $\angle ADB = 90^\circ$, $\therefore \angle BDC = 90^\circ$. 在 $\text{Rt} \triangle BDC$ 中, $\sin C = \frac{BD}{BC} = \frac{2\sqrt{2}}{\sqrt{26}} = \frac{2\sqrt{13}}{13}$.

$$\therefore EF = \frac{1}{2}AC = 2. \text{ 故选 A.}$$

3. k 【解析】如图. $\because AB = BC, \therefore \angle BAC = \angle C. \because DA \perp AB, DE \perp CA, \therefore \angle BAD = \angle DEA = 90^\circ, \therefore \angle BAC + \angle DAE = \angle DAE + \angle ADE = 90^\circ, \therefore \angle BAC = \angle ADE, \therefore \angle C = \angle ADE, \therefore \tan \angle ADE = \tan C = k.$
在 $\text{Rt} \triangle CDE$ 中, $\tan C = \frac{DE}{CE} = k, \therefore$ 令 $DE = mk, CE = m.$ 又 $\because CD = 1, \therefore m^2 + m^2 k^2 = 1^2, \therefore m^2 = \frac{1}{k^2 + 1}.$ 在 $\text{Rt} \triangle ADE$ 中, $\tan \angle ADE = \frac{AE}{DE} = k, \therefore AE = mk^2, \therefore AD = \sqrt{m^2 k^4 + m^2 k^2} = \sqrt{m^2 k^2 (k^2 + 1)} = \sqrt{\frac{1}{k^2 + 1} \cdot k^2 (k^2 + 1)} = k.$

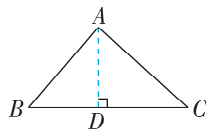
4. $\left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$ 【解析】如图所示, 将 OA 绕原点 O 顺时针旋转 75° 得到 $OA',$ 过 A' 作 $A'E \perp x$ 轴于 $E,$ 则 $OA' = OA = 3, \angle A'OE = 75^\circ - 30^\circ = 45^\circ, \therefore \triangle A'OE$ 是等腰直角三角形, $\therefore A'E = OE = OA' \times \cos 45^\circ = \frac{3\sqrt{2}}{2}.$
 $\because$ 点 A' 在第四象限, $\therefore$ 点 A 的对应点 A' 的坐标为 $\left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right).$ 故答案为 $\left(\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right).$

5. 2 【解析】 $\because AD \perp BC, \therefore \angle ADC = \angle ADB = 90^\circ. \because AC = 6\sqrt{2}, \angle C = 45^\circ, \therefore AD = AC \cdot \sin 45^\circ = 6\sqrt{2} \times \frac{\sqrt{2}}{2} = 6. \because \tan \angle ABC = 3, \therefore \frac{AD}{BD} = 3, \therefore BD = \frac{6}{3} = 2.$ 故答案为 2.

6. 【解】 (1) $\because \angle C = 90^\circ, AC = 4, \tan \angle ADC = 2, \therefore \frac{AC}{DC} = 2, \therefore DC = 2.$
(2) $\because BD - CD = 2, DC = 2, \therefore BD = 4, \therefore BC = BD + CD = 6, \therefore$ 在 $\text{Rt} \triangle ABC$ 中, $AB = \sqrt{BC^2 + AC^2} = \sqrt{6^2 + 4^2} = 2\sqrt{13}, \therefore \sin B = \frac{AC}{AB} = \frac{4}{2\sqrt{13}} = \frac{2\sqrt{13}}{13}.$

7. A 【解析】如图, 过点 A 作 $AD \perp BC,$ 垂足为 $D. \because$ 在 $\triangle ABC$ 中, $\angle BAC = 88^\circ, \angle C = 42^\circ,$

$\therefore \angle B = 180^\circ - 88^\circ - 42^\circ = 50^\circ. \because$ 在 $\text{Rt} \triangle ADB$ 中, $\sin B = \frac{AD}{AB}, AB = 6, \therefore AD = AB \cdot \sin B = 6\sin 50^\circ, \therefore$ 点 A 到 BC 的距离为 $6\sin 50^\circ.$ 故选 A.

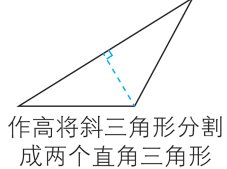
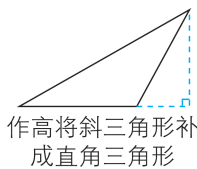


关键点拨

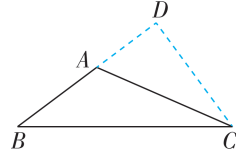
过点 A 作 $AD \perp BC$ 于点 $D,$ 构造出 $\text{Rt} \triangle ADB$ 是解题的关键.

8. B

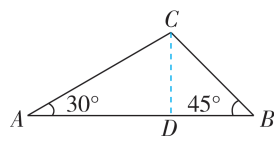
添加辅助线 构造直角三角形的一般方法



【解析】如图, 过点 C 作 $CD \perp AB,$ 交 BA 的延长线于点 $D. \because \angle BAC = 120^\circ, \therefore \angle DAC = 180^\circ - 120^\circ = 60^\circ, \therefore AD = AC \cdot \cos 60^\circ = 6 \times \frac{1}{2} = 3, CD = AC \cdot \sin 60^\circ = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}, \therefore BD = AB + AD = 7, \therefore BC = \sqrt{BD^2 + CD^2} = 2\sqrt{19}.$ 故选 B.



9. 【解】 (1) 如图, 过点 C 作 $CD \perp AB$ 于点 $D.$ 在 $\text{Rt} \triangle BCD$ 中, $\angle B = 45^\circ, BC = 3\sqrt{2}, \therefore BD =$



$BC \cdot \cos 45^\circ = 3\sqrt{2} \times \frac{\sqrt{2}}{2} = 3, \therefore CD = BD = 3.$
在 $\text{Rt} \triangle ACD$ 中, $\angle A = 30^\circ, \therefore AC = 6.$
(2) 由 (1) 知, 在 $\text{Rt} \triangle ACD$ 中, $AC = 6, CD = 3, \therefore AD = \sqrt{6^2 - 3^2} = 3\sqrt{3}, \therefore AB = AD + BD = 3\sqrt{3} + 3, \therefore S_{\triangle ABC} = \frac{1}{2}AB \cdot CD = \frac{9\sqrt{3} + 9}{2}.$

刷易错

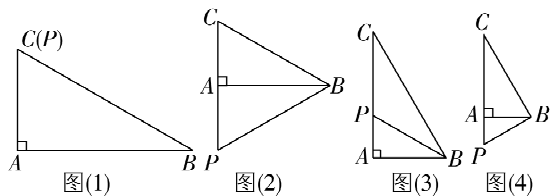
易错警示

本题没有明确三角形的哪个角是 $60^\circ,$ 故根据题意画出图形, 分 4 种情况进行讨论.

10. 6 或 $2\sqrt{3}$ 或 $4\sqrt{3}$ 【解析】①如图 (1), 当 $\angle C = 60^\circ,$ 点 P 在线段 AC 上时, $\angle ABC = 30^\circ,$ 此时点 P 与点 C 重合, $\therefore$ 此种情况不成立. ②如图 (2), 当 $\angle C = 60^\circ,$ 点 P 在线段 CA 的延长线上时, $\angle ABC = 30^\circ. \because \angle ABP = 30^\circ, \therefore \angle CBP = 60^\circ, \therefore \triangle PBC$ 是等边三角形, $\therefore CP = BC = 6.$ ③如图 (3), 当 $\angle ABC = 60^\circ,$ 点 P 在线段 AC 上时, $\angle C = 30^\circ. \because \angle ABP = 30^\circ, \therefore \angle PBC = 60^\circ - 30^\circ = 30^\circ = \angle C, \therefore PC = PB. \because BC = 6, \angle C = 30^\circ, \therefore AB = 3, \therefore PC =$

$$PB = \frac{AB}{\cos 30^\circ} = \frac{3}{\cos 30^\circ} = \frac{3}{\frac{\sqrt{3}}{2}} = 2\sqrt{3}. \quad \textcircled{4} \text{ 如}$$

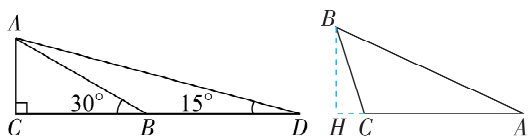
图(4), 当 $\angle ABC = 60^\circ$, 点 P 在线段 CA 的延长线上时, $\angle C = 30^\circ$. $\therefore \angle ABP = 30^\circ$, $\therefore \angle PBC = 60^\circ + 30^\circ = 90^\circ$, $\therefore PC = \frac{BC}{\cos 30^\circ} = 4\sqrt{3}$. 故答案为 6 或 $2\sqrt{3}$ 或 $4\sqrt{3}$.



刷提升

1. C 【解析】如图, 延长 AD, BC 交于点 E . $\therefore$ 四边形 $ABCD$ 是 $\odot O$ 的内接四边形, $\angle B = 90^\circ$, $\angle BCD = 120^\circ$, $\therefore \angle A = 60^\circ$, $\angle ADC = 90^\circ$, $\therefore \angle E = 30^\circ$, $\angle EDC = 90^\circ$. 在 $\text{Rt} \triangle CDE$ 中, $\tan 30^\circ = \frac{DC}{DE}$, $DC = 3$, $\therefore DE = 3\sqrt{3}$. 在 $\text{Rt} \triangle ABE$ 中, $\sin 30^\circ = \frac{AB}{AE}$, $AB = 5$, $\therefore AE = 10$, $\therefore AD = AE - DE = 10 - 3\sqrt{3}$. 故选 C.

2. B 【解析】如图, 在 $\text{Rt} \triangle ACB$ 中, $\angle C = 90^\circ$, $\angle ABC = 30^\circ$, 延长 CB 至点 D , 使 $BD = AB$, 连接 AD , 则 $\angle D = 15^\circ$. 设 $AC = 1$, 则 $BA = BD = 2$, $BC = \sqrt{AB^2 - AC^2} = \sqrt{3}$, $\therefore CD = BC + BD = 2 + \sqrt{3}$, $\therefore$ 在 $\text{Rt} \triangle ACD$ 中, $\tan 15^\circ = \tan D = \frac{AC}{CD} = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3}$. 故选 B.



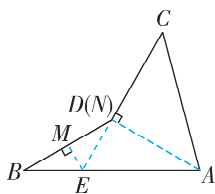
(第2题图)

(第3题图)

3. $\sqrt{13}$ 【解析】如图, 过 B 作 $BH \perp AC$, 交 AC 的延长线于 H , $\therefore BH \leq BC$. $\because AB = 4, BC = \sqrt{3}$, $\therefore BC < AB$, $\therefore \angle BAC$ 是锐角, 当 $\sin \angle BAC$ 最大时, $\angle BAC$ 最大. $\therefore \sin \angle BAC = \frac{BH}{AB}$, $AB = 4$, $BH \leq BC$, $\therefore$ 当 $BH = BC$, 即 $BC \perp AC$ 时, $\sin \angle BAC$ 最大, 此时 $\angle BAC$ 最大, $\therefore AC =$

$\sqrt{AB^2 - BC^2} = \sqrt{13}$. 故答案为 $\sqrt{13}$.

4. $5\sqrt{2}$ 【解析】如图, 延长 CD 交 AB 于点 E , 过点 E 作 $EM \perp BD$ 于点 M , 过点 A 作 $AN \perp CE$ 于点 N . $\because \angle BDC = 150^\circ$, $\therefore \angle BDE = 180^\circ - 150^\circ = 30^\circ$. $\because \angle B = 30^\circ$, $\therefore \angle B = \angle BDE$, $\therefore \angle AED = 60^\circ$, $BE = DE$. $\because EM \perp BD$, $\therefore DM = BM = \frac{1}{2}BD = \frac{5}{2}$, $\therefore$ 在 $\text{Rt} \triangle DEM$ 中, $DE = \frac{DM}{\cos \angle EDM} = \frac{DM}{\cos 30^\circ} = \frac{\frac{5}{2}}{\frac{\sqrt{3}}{2}} = \frac{5\sqrt{3}}{3}$, $\therefore CE = DE +$

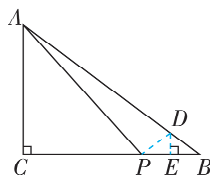


$CD = \frac{5\sqrt{3}}{3} + 5$. 设 $CN = x$. $\because \angle C = 45^\circ$, $\therefore AN = CN = x$, $AC = \sqrt{2}x$, $\therefore EN = CE - CN = 5 + \frac{5\sqrt{3}}{3} - x$. 在 $\text{Rt} \triangle AEN$ 中, $\angle AEN = 60^\circ$, $\therefore \frac{AN}{EN} = \frac{x}{5 + \frac{5\sqrt{3}}{3} - x} = \tan 60^\circ = \sqrt{3}$, 解得 $x = 5$, $\therefore$ 点 N 与

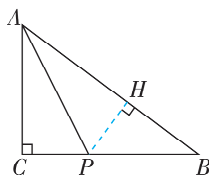
点 D 重合, $AC = \sqrt{2}x = 5\sqrt{2}$. 故答案为 $5\sqrt{2}$.

5. $\frac{7}{2}$ 或 5 【解析】当 $2\angle B + \angle BAP = 90^\circ$ 时, 如图(1), 在 AB 上取一点 D , 连接 DP , 使得 $DB = DP$, 过点 D 作 $DE \perp BC$ 于 E , $\therefore \angle DPB = \angle B$, $BP = 2BE$, $\therefore \angle ADP = \angle DPB + \angle B = 2\angle B$, $\therefore \angle ADP + \angle BAP = 90^\circ$, $\therefore \angle APD = 90^\circ$. $\therefore$ 在 $\text{Rt} \triangle ABC$ 中, $\angle C = 90^\circ$, $AC = 6, BC = 8$, $\therefore AB = \sqrt{AC^2 + BC^2} = 10$, $\therefore \cos B = \frac{BC}{AB} = \frac{4}{5}$, $\therefore$ 在

$\text{Rt} \triangle BDE$ 中, $\cos B = \frac{BE}{BD} = \frac{4}{5}$, 设 $BE = 4x$, 则 $BD = PD = 5x, BP = 8x$, $\therefore AD = AB - BD = 10 - 5x$, $PC = BC - BP = 8 - 8x$. 由勾股定理得 $AP^2 = AD^2 - PD^2 = AC^2 + CP^2$, $\therefore (10 - 5x)^2 - (5x)^2 = 6^2 + (8 - 8x)^2$, 解得 $x = \frac{7}{16}$ 或 $x = 0$ (舍去), $\therefore BP = 8x = \frac{7}{2}$.



图(1)



图(2)

易错警示
注意分两种情况讨论:

- ① $2\angle B + \angle BAP = 90^\circ$ 时,
- ② $2\angle BAP + \angle B = 90^\circ$ 时, 不能漏解.

关键点拨
理解题目中构造图形的方法是解题的关键.

当 $2\angle BAP + \angle B = 90^\circ$ 时, 如图(2). 过点 P 作 $PH \perp AB$ 于 H . $\therefore \angle BAC + \angle B = 90^\circ$, $\therefore \angle BAC = 2\angle BAP$, $\therefore AP$ 平分 $\angle BAC$, $\therefore PH = PC$. 设 $BP = 5m$, 则 $PH = CP = 8 - 5m$. $\therefore \sin B = \frac{PH}{PB} = \frac{AC}{AB} = \frac{3}{5}$, $\therefore PH = \frac{3}{5}PB = 3m$, $\therefore 8 - 5m = 3m$, $\therefore m = 1$, $\therefore BP = 5$. 综上所述, BP 的长为 $\frac{7}{2}$ 或 5 .

6. 【解】(1) $\because PF \perp AB$, $\therefore \angle AFP = \angle BFP = 90^\circ$.

$\because$ 四边形 $ABCD$ 是矩形,

$\therefore \angle A = 90^\circ = \angle BFP$, $\therefore AD \parallel FP$,

$\therefore \angle AEF = \angle PFG$.

$\because AE = 2, AF = x = 4$, $\therefore EF = \sqrt{2^2 + 4^2} = 2\sqrt{5}$.

$\because \frac{GF}{EF} = k = \frac{1}{2}$, $\therefore FG = \frac{1}{2}EF = \sqrt{5}$.

$\because \cos \angle PFG = \cos \angle AEF$, $\therefore \frac{AE}{EF} = \frac{FG}{PF}$, $\therefore \frac{2}{2\sqrt{5}} =$

$\frac{\sqrt{5}}{PF}$, $\therefore PF = 5$, 即 y 的值是 5. 故答案为 5.

(2) 由(1)可知 $\angle PFG = \angle AEF$. $\because PG \perp EF$, $\therefore \angle PGF = 90^\circ$, $\therefore \angle A = \angle PGF$, $\therefore \triangle PGF \sim$

$\triangle FAE$, $\therefore \frac{PF}{FE} = \frac{GF}{AE}$, $\therefore GF \cdot EF = PF \cdot AE$.

在 $\text{Rt} \triangle EAF$ 中, $\because AE = 2, AF = x$, $\therefore EF^2 = AE^2 + AF^2 = 2^2 + x^2 = 4 + x^2$.

$\because \frac{GF}{FE} = k$, $\therefore GF = kFE$, $\therefore kEF^2 = PF \cdot AE$,

即 $k(4 + x^2) = 2y$, $\therefore y = \frac{1}{2}k(x^2 + 4) = \frac{1}{2}kx^2 + 2k$.

(3) $\frac{3}{26} \leq k \leq 1$.

$\because$ 线段 CD 上存在点 P , $\therefore y = 6$, 即 $6 = \frac{1}{2}k(x^2 +$

$4)$, 则 $k = \frac{12}{x^2 + 4}$.

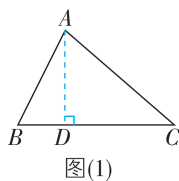
$\because 0 \leq x \leq 10$, $\therefore 4 \leq x^2 + 4 \leq 104$, $\therefore \frac{3}{26} \leq k \leq 3$.

$\because \frac{GF}{FE} = k$, 点 G 在 EF 上, $\therefore k \leq 1$, $\therefore \frac{3}{26} \leq k \leq 1$.

刷素养

7. (1) 【证明】如图(1), 过点 A 作 $AD \perp BC$ 于 D .

在 $\text{Rt} \triangle ADB$ 中, $\sin \angle ABC = \frac{AD}{AB}$,



图(1)

思路分析

(1) 根据题意得出 $\angle AEF = \angle PFG$, 则

$\cos \angle PFG = \cos \angle AEF$, 代入计算即可;

(2) 根据题目条件证明 $\triangle PGF \sim \triangle FAE$, 得

$\frac{PF}{FE} = \frac{GF}{AE}$, 再

由 $\frac{GF}{FE} = k$, 得

$GF = kFE$, 进而可得答案;

(3) 根据点 P 在 CD 上, 可得 $y = 6$, 进而

得 $k = \frac{12}{x^2 + 4}$, 由

$0 \leq x \leq 10$ 得

$\frac{3}{26} \leq k \leq 3$, 再

由点 G 在 EF 上, 可得 $k \leq 1$, 进而解决问题.

在 $\text{Rt} \triangle ACD$ 中, $\sin \angle ACB = \frac{AD}{AC}$, $\therefore \sin \angle ABC =$

$\sin \angle ACB = \frac{AD}{AB} \cdot \frac{AB}{AC} = AC : AB$.

同理可得 $\sin \angle BAC : \sin \angle ACB = BC : AB$, $\therefore BC : AC : AB = \sin \angle BAC : \sin \angle ACB : \sin \angle ABC$.

(2) 【解】 $\frac{\sin B}{\sin \angle ACB} = \frac{5}{6}$.

如图(2), 过点 C 作 CE 平分 $\angle ACB$, 交 AB 于 E .

$\because AC = \frac{5}{4}BC$, $\therefore$ 设 $BC = 4x$, 则

$AC = 5x$.

$\because CE$ 平分 $\angle ACB$, $\therefore \angle ACE = \angle BCE$.

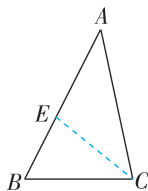
$\because \angle ACB = 2\angle A$, $\therefore \angle ACE = \angle A = \angle BCE$, $\therefore AE = CE$.

又 $\because \angle B = \angle B$, $\therefore \triangle ABC \sim \triangle CBE$, $\therefore \frac{BC}{AB} =$

$\frac{CE}{AC} = \frac{BE}{BC}$, $\therefore \frac{4x}{AB} = \frac{CE}{5x} = \frac{BE}{4x}$, $\therefore BE = \frac{4}{5}CE$, $\therefore BE =$

$\frac{4}{5}AE$, $\therefore AB = \frac{9}{5}AE$, $\therefore 4x \cdot 5x = \frac{9}{5}AE^2$, $\therefore AE =$

$\frac{10}{3}x$, $\therefore AB = 6x$, $\therefore \frac{\sin B}{\sin \angle ACB} = \frac{AC}{AB} = \frac{5}{6}$.



图(2)

8.3.2 实际问题

课时1 仰角、俯角、坡度、坡角问题

刷基础

1. B 【解析】 $\because AB = a, AB \perp CD, \angle BAD = \beta, \angle BAC = \alpha$, $\therefore$ 在 $\text{Rt} \triangle ABD$ 中, $BD = AB \cdot \tan \beta = a \tan \beta$, 在 $\text{Rt} \triangle ABC$ 中, $BC = AB \cdot \tan \alpha = a \tan \alpha$, $\therefore CD = BD + BC = a \tan \beta + a \tan \alpha$. 故选 B.

2. 16 【解析】由题意得 $AD \perp BD, BC = 12$ 米. 设 $CD = x$ 米, 则 $BD = BC + CD = (x + 12)$ 米. 在 $\text{Rt} \triangle ABD$ 中, $\angle B = 30^\circ$, $\therefore AD = BD \cdot \tan 30^\circ = \frac{\sqrt{3}}{3}(x + 12)$ 米. 在 $\text{Rt} \triangle ACD$ 中, $\angle ACD = 45^\circ$, $\therefore AD = CD \cdot \tan 45^\circ = x$ 米, $\therefore x = \frac{\sqrt{3}}{3}(x + 12)$,

解得 $x = 6\sqrt{3} + 6$, $\therefore AD = 6\sqrt{3} + 6 \approx 6 \times 1.73 + 6 \approx 16$ (米). 故答案为 16.

3. A 【解析】 $\because$ 斜坡的坡比为 $1:2$, $\therefore$ 设斜坡上最高点离地面的高度为 h 米, 则水平距离为 $2h$ 米. $\because$ 斜坡长 5 米, $\therefore h^2 + (2h)^2 = 5^2$, 解得

$h=\sqrt{5}$ (负值已舍去), 即斜坡上最高点离地面的高度为 $\sqrt{5}$ 米. 故选 A.

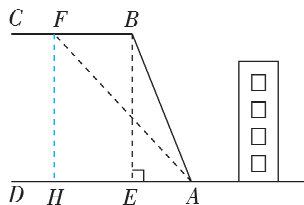
4. $(3\sqrt{3}-3)$ 【解析】由题意可得 $AC=3$ 米, $\angle ABC=45^\circ$, $\angle ADC=30^\circ$, $\therefore BC=\frac{AC}{\tan 45^\circ}=\frac{3}{1}=3$ (米), $CD=\frac{AC}{\tan 30^\circ}=\frac{3}{\frac{\sqrt{3}}{3}}=3\sqrt{3}$ (米), $\therefore BD=CD-BC=(3\sqrt{3}-3)$ 米. 故答案为 $(3\sqrt{3}-3)$.

5. $80\sqrt{17}$ 米 【解析】 $\because \angle AEB=90^\circ$, $AB=200$ 米, 斜坡 AB 的坡度为 $1:\sqrt{3}$, $\therefore \tan \angle ABE=\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$, $\therefore \angle ABE=30^\circ$, $\therefore AE=\frac{1}{2}AB=100$ 米. $\because AC=20$ 米, $\therefore CE=80$ 米. $\because$ 斜坡 CD 的坡度为 $1:4$, $\therefore \frac{CE}{DE}=\frac{1}{4}$, 即 $\frac{80}{DE}=\frac{1}{4}$, $\therefore DE=320$ 米, $\therefore CD=\sqrt{CE^2+DE^2}=\sqrt{80^2+320^2}=80\sqrt{17}$ (米). 故答案为 $80\sqrt{17}$ 米.

6. 【解】(1) $\because$ 斜坡 AB 的坡度 $i=1:\frac{5}{12}$, $\therefore BE:EA=12:5$, $\therefore$ 设 $BE=12x$ 米, 则 $EA=5x$ 米. 由勾股定理得 $BE^2+EA^2=AB^2$, 即 $(12x)^2+(5x)^2=26^2$, 解得 $x=2$ (负值已舍去), 则 $BE=12x=24$.

答: 改造前坡顶与地面的距离 BE 的长为 24 米.

- (2) 如图, 作 $FH \perp AD$ 于 H , 易知四边形 $FBEH$ 是矩形, $\therefore FB=HE$, $FH=BE=24$.



由(1)得 $AE=5x=10$.

由题意得当 BF 最短时, $\angle FAH=53^\circ$.

$$\because \tan \angle FAH = \frac{FH}{AH}, \therefore AH = \frac{FH}{\tan \angle FAH} = \frac{24}{\tan 53^\circ} \approx \frac{24}{1.33} \approx 18, \therefore BF=HE=AH-AE=18-10=8 \text{ (米)}.$$

答: BF 至少是 8 米.



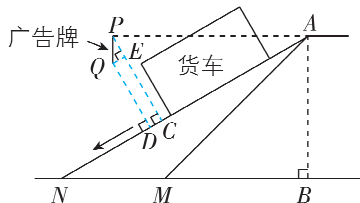
刷提升

1. B 【解析】在 $\text{Rt} \triangle CDE$ 中, $\because CD=10$ m, $DE=5$ m, $\therefore \sin \angle DCE = \frac{DE}{CD} = \frac{5}{10} = \frac{1}{2}$, $\therefore \angle DCE = 30^\circ$. $\because \angle ACB = 60^\circ$, $\therefore \angle ABC = 30^\circ$, $\angle DCB = 90^\circ$. $\because \angle BDF = 30^\circ$, $\therefore \angle DBF = 60^\circ$, $\therefore \angle DBC = 30^\circ$, $\therefore BC = \frac{CD}{\tan 30^\circ} = \frac{10}{\frac{\sqrt{3}}{3}} = 10\sqrt{3}$ (m), $\therefore AB = BC \cdot \sin 60^\circ = 10\sqrt{3} \times \frac{\sqrt{3}}{2} = 15$ (m). 故选 B.

2. 【解】(1) $\because AB \perp BN$, $\therefore \angle ABM = 90^\circ$. $\because \angle AMB = 45^\circ$, $\therefore \angle MAB = 90^\circ - 45^\circ = 45^\circ$, $\therefore \triangle ABM$ 是等腰直角三角形, $\therefore AB=BM$. $\because AB=5$ 米, $\therefore BM=5$ 米. $\because \angle ANB = 30^\circ$, $\therefore$ 在 $\text{Rt} \triangle ANB$ 中, $BN = \frac{AB}{\tan \angle ANB} = 5\sqrt{3}$ (米), $\therefore MN = BN - BM = 5\sqrt{3} - 5 \approx 5 \times 1.73 - 5 \approx 3.7$ (米).

答: 坡底部增加的长度 MN 是 3.7 米.

- (2) 如图, 过点 P 作 $PC \perp AN$, 垂足为 C , 过点 Q 作 $QD \perp AN$, 垂足为 D , 过点 Q 作 $QE \perp PC$, 垂足为 E ,



则 $\angle QDC = \angle ECD = \angle QEC = 90^\circ$, $\therefore$ 四边形 $QDCE$ 是矩形, $\therefore QD=EC$.

由题知 $AP \parallel BN$, $\angle ANB = 30^\circ$, $\therefore \angle PAC = \angle ANB = 30^\circ$, $\therefore \angle APC = 90^\circ - 30^\circ = 60^\circ$.

$\because AP \parallel BN$, $PQ \perp BN$, $\therefore AP \perp PQ$, $\therefore \angle APQ = 90^\circ$, $\therefore \angle QPE = \angle APQ - \angle APC = 30^\circ$.

$\because AP=7.5$ 米, $\therefore$ 在 $\text{Rt} \triangle APC$ 中, $PC = \frac{1}{2}AP = 3.75$ 米.

$\because PQ=0.7$ 米, $\therefore$ 在 $\text{Rt} \triangle PQE$ 中, $PE =$

$$PQ \cdot \cos \angle QPE = 0.7 \times \frac{\sqrt{3}}{2} = \frac{7\sqrt{3}}{20} \text{ (米)},$$

$$\therefore QD=EC=PC-PE=3.75-\frac{7\sqrt{3}}{20} \approx 3.1 \text{ (米)}.$$

关键点拨

根据坡比为 1:2 设出相应的未知数, 再根据斜坡长为 5 米及勾股定理列出方程是解题的关键.

关键点拨

本题考查的是解直角三角形的应用——坡度、坡角问题, 掌握坡度是坡面的铅直高度 h 和水平宽度 l 的比是解题的关键.

课时 2 其他问题

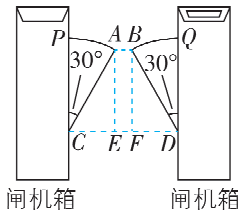


刷基础

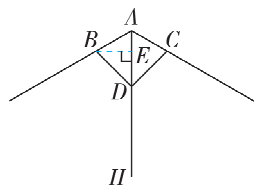
1. B 【解析】在 $\text{Rt} \triangle ABC$ 中, $\angle BAC = \alpha$, $AC = 5$ 米, $\therefore BC = AC \cdot \tan \alpha = 5 \tan \alpha$ 米, $\therefore$ 所需地毯的长度为 $BC + AC = (5 \tan \alpha + 5)$ 米. 故选 B.

2. 2.9 【解析】在 $\text{Rt} \triangle AMD$ 中, $AM = 4$ 米, $\angle MAD = 45^\circ$, $\therefore DM = 4$ 米. 又 $AB = 8$ 米, $\therefore MB = 12$ 米. 在 $\text{Rt} \triangle CMB$ 中, $\angle MBC = 30^\circ$, $\therefore MC = BM \cdot \tan 30^\circ = 12 \times \frac{\sqrt{3}}{3} = 4\sqrt{3}$ (米), $\therefore DC = MC - MD = 4\sqrt{3} - 4 \approx 2.9$ (米). 故答案为 2.9.

3. 15 【解析】如图, 连接 AB, CD , 过点 A 作 $AE \perp CD$ 于 E , 过点 B 作 $BF \perp CD$ 于 F . 易知四边形 $AEFB$ 是矩形, $\therefore EF = AB$. $\therefore$ 由题可知 $AE \parallel PC$, $\therefore \angle PCA = \angle CAE = 30^\circ$, $\therefore CE = AC \cdot \sin 30^\circ = 55 \times \frac{1}{2} = 27.5$ (cm). 同理可得 $DF = 27.5$ cm. $\therefore CD = 70$ cm, $\therefore AB = EF = CD - CE - DF = 70 - 27.5 - 27.5 = 15$ (cm).



(第 3 题图)



(第 4 题图)

4. 【解】如图, 过点 B 作 $BE \perp AD$ 于点 E . $\therefore \angle BAC = 120^\circ$, AH 平分 $\angle BAC$, $\therefore \angle BAD = \angle CAD = \frac{1}{2} \angle BAC = 60^\circ$, $\therefore AE = AB \times \cos 60^\circ = 20 \times \frac{1}{2} = 10$ (cm), $BE = AB \times \sin 60^\circ = 20 \times \frac{\sqrt{3}}{2} \approx 17.32$ (cm).

$\therefore AB = AC$, $\angle BAD = \angle CAD$, $AD = AD$, $\therefore \triangle ABD \cong \triangle ACD$ (SAS), $\therefore \angle ADB = \angle ADC$. $\therefore \angle BDC = 90^\circ$, $\therefore \angle BDE = 45^\circ$, $\therefore DE = BE = 17.32$ cm, $\therefore AD = AE + DE = 10 + 17.32 = 27.32$ (cm).

$\therefore \frac{DH}{AH} \approx 0.618$, $\therefore \frac{AH - AD}{AH} \approx 0.618$, $\therefore AH \approx 72$ cm.

经检验, $AH \approx 72$ 是分式方程的解. 答: 最少需要准备 72 cm 长的伞柄.

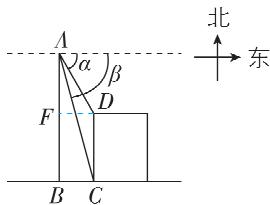
$\therefore 3.1 > 3$, $\therefore$ 高度为 3 米的货车沿 AN 下坡过程中不会撞到广告牌.

答: 广告牌下端点 Q 到斜坡 AN 的距离约为 3.1 米, 高度为 3 米的货车沿 AN 下坡过程中不会撞到广告牌.

刷素养

3. 【解】(1) $\cos 75^\circ = \cos (45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$.

(2) 如图(1), 过点 D 作 $DF \perp AB$ 于点 F , 易知四边形 $CDFB$ 是矩形, $\therefore CD = BF$.



图(1)

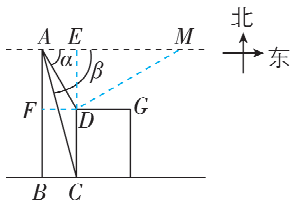
由题可知 $\angle ACB = \beta = 75^\circ$, $BC = 42$ m, $\therefore AB = BC \cdot \tan \angle ACB = 42 \times \tan 75^\circ = 42 \times$

$$\frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \cdot \tan 30^\circ} = 42 \times \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = 42 (\sqrt{3} + 1)$$

2) m. 由题知 $\angle ADF = \alpha = 60^\circ$, $DF = BC = 42$ m, $\therefore AF = DF \cdot \tan \alpha = 42\sqrt{3}$ m, $\therefore CD = BF = AB - AF = 84$ m.

答: 建筑物 CD 的高为 84 m.

(3) 如图(2), 过点 D 作 $DF \perp AB$ 于 F , 延长 CD 交 AE 于点 E , 作 $\angle GDM$ 交直线 AE 于点 M , 使 $\angle GDM = 30^\circ$.



图(2)

易知 $AE = BC = 42$ m.

由(2)易得 $DE = AF = 42\sqrt{3}$ m.

$\therefore \angle EDM = 90^\circ - \angle GDM = 60^\circ$,

$\therefore EM = \tan 60^\circ \cdot DE = 126$ m, $\therefore AM = AE + EM = 42 + 126 = 168$ (m).

答: 飞机从点 A 处往正东方向飞 168 m, 居民在点 D 处看飞机的仰角恰好为 30° .

思路分析

(2) 过点 D 作 $DF \perp AB$ 于点 F , 求出 AF 和 AB 的长即可得出答案;

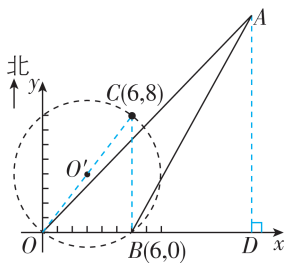
(3) 延长 CD 交 AE 于点 E , 作 $\angle GDM$ 交 AE 于点 M , 并使 $\angle GDM = 30^\circ$, 根据 60° 角的正切可得 EM , 再计算 AM 即可.

关键点拨

过点 B 作 $BE \perp AD$ 于点 E , 构造出 $\text{Rt} \triangle ABE$ 是解题的关键.

5. $10\sqrt{6}$ 【解析】如图,作 $BD \perp AC$ 于点 D . 由题意得 $\angle CBA = 25^\circ + 50^\circ = 75^\circ$, $AB = 20$, $\angle CAB = (90^\circ - 70^\circ) + (90^\circ - 50^\circ) = 20^\circ + 40^\circ = 60^\circ$, $\therefore \angle ABD = 30^\circ$, $\therefore \angle CBD = 75^\circ - 30^\circ = 45^\circ$. 在 $\text{Rt} \triangle ABD$ 中, $BD = AB \cdot \sin \angle CAB = 20 \times \sin 60^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$. 在 $\text{Rt} \triangle BCD$ 中, $\angle CBD = 45^\circ$, $\therefore BC = \sqrt{2} BD = 10\sqrt{3} \times \sqrt{2} = 10\sqrt{6}$ (海里). 故答案为 $10\sqrt{6}$.

6. 【解】(1) 如图, 连接 CB, CO , 则 $CB \parallel y$ 轴, $\therefore \angle CBO = 90^\circ$,



$\therefore$ 可设 OC 的中点 O' 为由 O, B, C 三点所确定的圆的圆心, 则 OC 为 $\odot O'$ 的直径. 由已知得 $OB = 6, CB = 8$, 由勾股定理得 $OC = \sqrt{8^2 + 6^2} = 10$, $\therefore$ 半径 $OO' = 5$, $\therefore S_{\odot O'} = 25\pi$, 即圆形区域的面积为 25π .

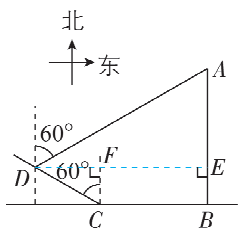
(2) 如图, 过点 A 作 $AD \perp x$ 轴于点 D . 依题意, 得 $\angle ABD = 61^\circ$. 设 $AD = x$. 在 $\text{Rt} \triangle ABD$ 中, $\tan \angle ABD = \frac{AD}{BD}$, $\therefore BD = \frac{x}{\tan 61^\circ}$. 由题意得 $\angle AOD = 45^\circ$, $\therefore AD = OD = x$. $\therefore OB = OD - BD$, $\therefore x - \frac{x}{\tan 61^\circ} = 6$, 解得 $x \approx 13.5$.

在 $\text{Rt} \triangle ABD$ 中, $\sin \angle ABD = \sin 61^\circ = \frac{AD}{AB}$, 即 $0.87 \approx \frac{13.5}{AB}$, $\therefore AB \approx 15.5$.

即观测点 B 到渔船 A 的距离约为 15.5 .

刷提升

1. C 【解析】如图所示, 过点 D 作 $DE \perp AB$ 于点 E , 过点 C 作 $CF \perp DE$ 于点 F , $\therefore$ 四边形 $BCFE$ 是矩形, $\therefore EF = BC = 15 \text{ km}, CF = BE$. 由题意



思路分析

(1) 根据题意可以求得圆的半径, 从而可求圆形区域的面积.

(2) 过点 A 作 $AD \perp x$ 轴于点 D , 设 $AD = x$. 在 $\text{Rt} \triangle ABD$ 中, $BD = \frac{x}{\tan 61^\circ}$. 由 $\angle AOD = 45^\circ$ 得 $AD = OD = x$, 由 $OB = OD - BD$ 可得 $x - \frac{x}{\tan 61^\circ} = 6$, 解方程求得 x , 进而解直角三角形求得 AB .

思路分析

过 D 作 $DM \perp AC$ 于 M , 过 B 作 $BN \perp AE$ 于 N , 设 $MD = x$. 在直角三角形中, 利用锐角三角函数即可用含 x 的式子表示出 AM 与 CM , 根据 $AC = AM + CM$ 列方程, 从而求得 MD 的长, 进一步求得 AD 的长, 再利用锐角三角函数求出 AN 与 NE 的长, 进而求得 DE .

得 $\angle DCF = 60^\circ$, $\therefore DF = \sin 60^\circ \times CD = \frac{\sqrt{3}}{2} \times 10 = 5\sqrt{3}$ (km), $BE = CF = \cos 60^\circ \times CD = \frac{1}{2} \times 10 = 5$ (km), $\therefore DE = DF + EF = (5\sqrt{3} + 15)$ km. 由题意得 $\angle ADE = 90^\circ - 60^\circ = 30^\circ$, $\therefore AE = DE \cdot \tan \angle ADE = \frac{\sqrt{3}}{3} \times (5\sqrt{3} + 15) = (5\sqrt{3} + 5)$ km, $\therefore AB = AE + BE = 5\sqrt{3} + 5 + 5 = (5\sqrt{3} + 10)$ km. 故选 C.

2. 27.5 【解析】如图, 过点 B 作 $BG \perp CF$ 于点 G , 作 $BH \perp DE$ 于点 H . 由题意可得 $\angle ABH = 10^\circ$. $\therefore AB = 24 \text{ cm}, BE = \frac{1}{3}AB$, $\therefore BE = 8 \text{ cm}$, $\therefore$ 在 $\text{Rt} \triangle BEH$ 中, $EH = BE \cdot \sin 10^\circ \approx 8 \times 0.17 = 1.36$ (cm), $BH = BE \cdot \cos 10^\circ \approx 8 \times 0.98 = 7.84$ (cm). $\therefore AC \perp CF, AC \parallel DE$, $\therefore DE \perp CF$. 又 $\because BH \perp DE, BG \perp CF$, $\therefore$ 四边形 $BGDH$ 是矩形, $\therefore DH = BG, DG = BH = 7.84 \text{ cm}$. $\therefore \angle BFC = 45^\circ, BG \perp CF, MN \perp CF$, $\therefore BG = GF, NF = MN = 8 \text{ cm}$. $\therefore DN = 26 \text{ cm}$, $\therefore GF = DN + NF - DG = 26 + 8 - 7.84 = 26.16$ (cm), $\therefore DH = BG = GF = 26.16 \text{ cm}$, $\therefore DE = DH + EH = 26.16 + 1.36 \approx 27.5$ (cm).

3. 580 【解析】如图, 过 D 作 $DM \perp AC$ 于 M , 设 $MD = x \text{ m}$. 在 $\text{Rt} \triangle MAD$ 中, $\angle MAD = 45^\circ$, $\therefore \triangle ADM$ 是等腰直角三角形, $\therefore AM = MD = x$, $\therefore AD = \sqrt{2}x$. 在 $\text{Rt} \triangle MCD$ 中, $\angle MDC = 63.4^\circ$, $\therefore MC = MD \cdot \tan 63.4^\circ \approx 2x$. $\therefore AC = 600 + 600 = 1200, AC = AM + MC$, $\therefore x + 2x = 1200$, 解得 $x = 400$, $\therefore MD = 400, AD = 400\sqrt{2}$. 过 B 作 $BN \perp AE$ 于 N . $\therefore \angle EAB = 45^\circ, \angle EBC = 90^\circ - 15^\circ = 75^\circ$, $\therefore \angle E = 30^\circ$. 在 $\text{Rt} \triangle ABN$ 中, $\angle NAB = 45^\circ, AB = 600$, $\therefore BN = AN = \frac{\sqrt{2}}{2}AB = 300\sqrt{2}$, $\therefore DN = AD - AN = 400\sqrt{2} - 300\sqrt{2} = 100\sqrt{2}$. 在 $\text{Rt} \triangle NBE$ 中, $\angle E = 30^\circ$, $\therefore NE = \sqrt{3}BN = 300\sqrt{6}$, $\therefore DE = NE - DN = 300\sqrt{6} - 100\sqrt{2} \approx 580$ (m).

